NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 17-03-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take g = 10 ms^{-2})

(1) 3.0 ms^{-1} (2) 3.50 ms^{-1} (3) 2.0 ms^{-1} (4) 2.50 ms^{-1}

Ans. (4)

Sol. $v_0 = \sqrt{2gh}$

$v = e\sqrt{2gh} = \sqrt{2gh}$

$\Rightarrow$ e = 0.9

$S = h + 2e^2h + 2e^4 h + ...$

$t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$

$v_{av} = \frac{s}{t} = 2.5 \text{ m/s}$

2. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is :

(1) 1.02 (2) 1.2 (3) 1.25 (4) 1.35

Ans. (2)

Sol. f = 4 + 3 + 3 = 10

assuming non linear

$\beta = \frac{C_P}{C_V} = 1 + \frac{2}{f} = \frac{12}{10} = 1.2$

3. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take In 2 = 0.693)

(1) 0.69×10^2 kg s^{-1} (2) 3.3×10^2 kg s^{-1} (3) 1.16×10^2 kg s^{-1} (4) 5.7×10^{-3} kg s^{-1}

Ans. (NA)

ZIGYAN Ans. (Bonus)

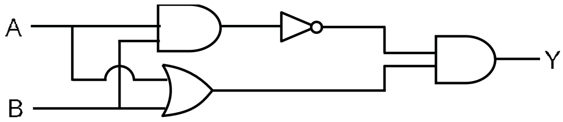
Sol. $A = A_0e^{-\gamma t}$

$\ln 2 = \frac{b}{2m} \times 120$

$\frac{0.693 \times 2 \times 1}{120} = b$

1.16×10^{-2} kg/sec.

4. Which one of the following will be the output of the given circuit ?



(1) NOR Gate (2) NAND Gate (3) AND Gate (4) XOR Gate

Ans. (4)

Sol. Conceptual

5. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be ___________.

[Given : density of water is 1000 kg m^{-3} and g = 9.8 ms^{-2}.]

(1) 1.96×10^7 Nm^{-2} (2) 1.44×10^7 Nm^{-2} (3) 2.26×10^9 Nm^{-2} (4) 1.44×10^9 Nm^{-2}

Ans. (4)

Sol. P = hρg

$$\beta = \frac{p}{\frac{\Delta V}{V}} = \frac{2 \times 10^3 \times 10^3 \times 9.8}{1.36 \times 10^{-2}}$$

$= 1.44 \times 10^9$ N/m^2

6. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P' , R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is _________.'P' has the time period of 24 hours.

(1) $6\sqrt{2}$ (2) $\frac{6}{\sqrt{2}}$ (3) 3 (4) 5

Ans. (3)

Sol. $T \propto R^{3/2}$

$$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3hr$$

7. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

(1) Y(x,t) = 0.03 [sin 5.1 x – (0.2×10^3)t]

(2) Y(x,t) = 0.06 [sin 5.1 x – (1.5×10^3)t]

(3) Y(x,t) = 0.06 [sin 0.8 x – (0.5×10^3)t]

(4) Y(x,t) = 0.03 [sin 5.1 x – (1.5×10^3)t]

Ans. (4)

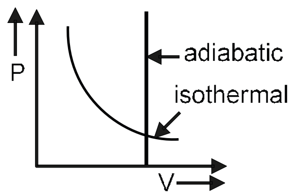
Sol. $\omega = 2\pi f$

$= 1.5 \times 10^3$

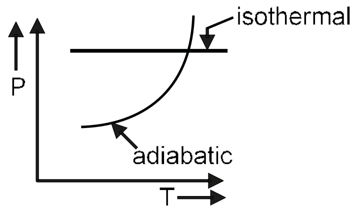
$A = \frac{6}{2} = 3 \text{ cm} = 0.03 \text{ m}$

8. Which one is the correct option for the two different thermodynamic processes ?

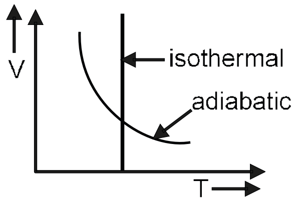
(a)



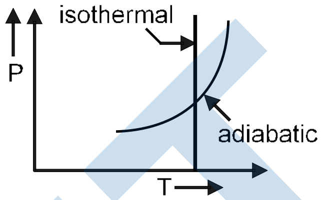
(b)



(c)



(d)



(1) (c) and (a) (2) (c) and (d) (3) (a) only (4)(b) and (c)

Ans. (2)

Sol. Option (a) is wrong ; since in adiabatic process $V \neq$ constant.

Option (b) is wrong, since in isothermal process T = constant

Option (c) & (d) matches isotherms & adiabatic formula :

$TV^{\gamma-1} = \text{constant}$ & $\frac{T^{\gamma}}{p^{\gamma-1}} = \text{constant}$

9. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is x = 0 at t = 0 ; then its displacement after time (t = 1) is :

(1) $v_0 + g + F$ (2) $v_0 + \frac{g}{2} + \frac{F}{3}$ (3) $v_0 + \frac{g}{2} + F$ (4) $v_0 + 2g + 3F$

Ans. (2)

Sol. $v = v_0 + gt + Ft^2$

$\frac{ds}{dt} = v_0 + gt + Ft^2$

$\int ds = \int_0^1 (V_0 + gt + Ft^2)dt$

$s = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_0^1$

$s = v_0 + \frac{g}{2} + \frac{F}{3}$

10. A carrier signal $C(t) = 25 \sin(2.512 \times 10^{10} t)$ is amplitude modulated by a message signal $m(t) = 5 \sin(1.57 \times 10^{8} t)$ and transmitted through an antenna. What will be the bandwidth of the modulated signal?

(1) 8 GHz (2) 2.01 GHz (3) 1987.5 MHz (4) 50 MHz

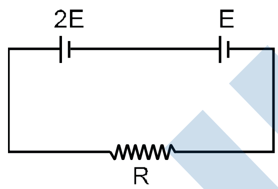
Ans. (4)

Sol. Band width = $2 f_m$

$\omega_m = 1.57 \times 10^{8} = 2\pi f_m$

$BW = 2f_m = \frac{10^8}{2} Hz = 50\ MHz$

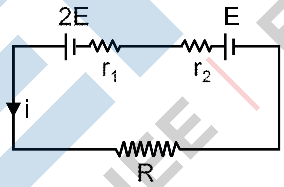
11. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is



(1) $r_1 + r_2$ (2) $\frac{r_1}{2} - r_2$ (3) $\frac{r_1}{2} + r_2$ (4) $r_1 - r_2$

Ans. (2)

Sol. $i = \frac{3E}{R + r_1 + r_2}$



$TPD = 2E - ir_1 = 0$

$2E = ir_1$

$2E = \frac{3E \times r_1}{R + r_1 + r_2}$

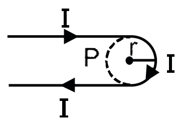
$2R + 2r_1 + 2r_2 = 3r_1$

$R = \frac{r_1}{2} - r_2$

12. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle ?

(1) $\frac{\mu_0 I}{4\pi r}(2 - \pi)$ (2) $\frac{\mu_0 I}{4\pi r}(2 + \pi)$

(3) $\frac{\mu_0 I}{2\pi r}(2 + \pi)$ (4) $\frac{\mu_0 I}{2\pi r}(2 - \pi)$



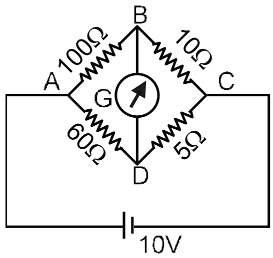
Ans. (2)

Sol. $B = 2 \times B_{st.\ wire} + B_{loop}$

$$B = 2 \times \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{2r}\left(\frac{\pi}{2\pi}\right)$$

$$B = \frac{\mu_0 i}{4\pi r}(2+\pi)$$

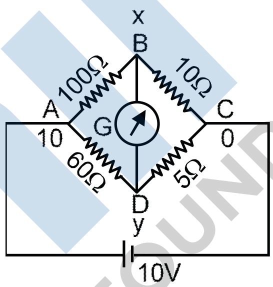
13. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC.



(1) 2.44 μA (2) 2.44 mA (3) 4.87 mA (4) 4.87 μA

Ans. (3)

Sol. $\frac{x-10}{100} + \frac{x-y}{15} + \frac{x-0}{10} = 0$



$53x - 20y = 30$ .....(1)

$\frac{y-10}{60} + \frac{y-x}{15} + \frac{y-0}{5} = 0$

$17y - 4x = 10$ ......(2)

on solving (1) & (2)

$x = 0.865$

$y = 0.792$

$\Delta V = 0.073$ R = 15Ω

i = 4.87 mA

14. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

(1) $\frac{K_2}{K_1}$ (2) $\frac{K_1}{K_2}$ (3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_2}{K_1}}$

Ans. (4)

Sol. $A_1\omega_1 = A_2\omega_2$

$$A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

15. Match List-I with List-II

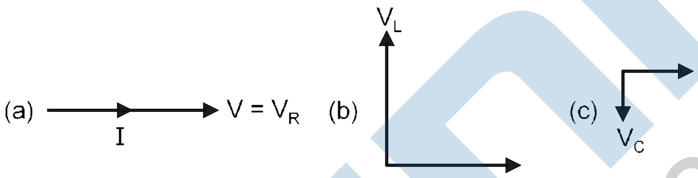
	List-I		**List-II**
(a)	Phase difference between current and voltage in a purely resistive AC circuit	(i)	$\frac{\pi}{2}$; current leads voltage
(b)	Phase difference between current and voltage in a pure inductive AC circuit	(ii)	zero
(c)	Phase difference between current and voltage in a pure capacitive AC circuit	(iii)	$\frac{\pi}{2}$; current lags voltage
(d)	Phase difference between current and voltage in an LCR series circuit	(iv)	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :

(1) (a)–(i),(b)–(iii),(c)–(iv),(d)–(ii)
(2) (a)–(ii),(b)–(iv),(c)–(iii),(d)–(i)
(3) (a)–(ii),(b)–(iii),(c)–(iv),(d)–(i)
(4) (a)–(ii),(b)–(iii),(c)–(i),(d)–(iv)

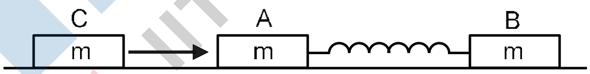
Ans. (4)

Sol.



(d) $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$

16. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is



(1) $v\sqrt{\frac{M}{2K}}$
(2) $\sqrt{\frac{mv}{2K}}$
(3) $\sqrt{\frac{mv}{K}}$
(4) $\sqrt{\frac{m}{2K}}$

Ans. (1)

Sol. C comes to rest

V_{cm} of A & B $= \frac{v}{2}$

$\Rightarrow \frac{1}{2} \text{ is } v_{net}^2 = \frac{1}{2}kx^2$

$x = \sqrt{\frac{\mu \times v^2}{k}} = \sqrt{\frac{m}{2k}}v$

17. The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region ?

(1) Brackett series (2) Paschen series (3) Lyman series (4) Balmer series

Ans. (4)

Sol. Conceptual

18. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

(1) $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$ (2) $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

(3) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$ (4) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

Ans. (1)

Sol. $\frac{1}{2}mv_1^2 = hf_1 - \phi$

$\frac{1}{2}mv_2^2 = hf_2 - \phi$

$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

19. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved ?

(1) Both, inductive reactance and current will be halved.

(2) Inductive reactance will be halved and current will be doubled.

(3) Inductive reactance will be doubled and current will be halved.

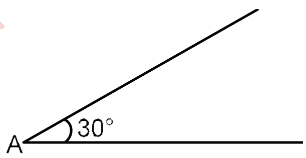
(4) Both, inducting reactance and current will be doubled.

Ans. (2)

Sol. (2) $X_L = \omega L$

$i = \frac{V_0}{\omega L}$

20. A sphere of mass 2kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How low will the sphere take to return to the starting point A ?



(1) 0.60 s (2) 0.52 s (3) 0.57 s (4) 0.80 s

Ans. (3)

Sol. $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{5}{7} \times \frac{10}{2} = \frac{25}{7}$

$t = \frac{2v_0}{a} = \frac{2 \times 1 \times 7}{25}$

= 0.56

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\frac{x}{5}}E$

Where the value of x =________.

Ans. (3)

Sol. $c \in_0 E^2 = \frac{100}{4\pi \times 3^2}$

$c \in_0 \left(\sqrt{\frac{x}{5}}E\right)^2 = \frac{60}{4\pi \times 3^2}$

$\Rightarrow \frac{x}{5} = \frac{3}{5}$

$\Rightarrow x = 3$

2. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be __________.

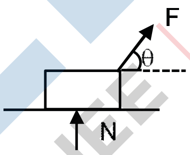
[Take g = 10 ms^{-2}]

Ans. (5)

Sol. F cos θ = mN

F sin θ + N = mg

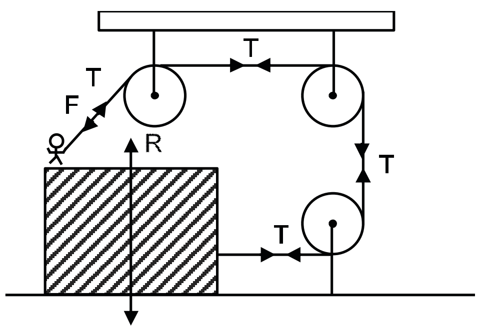
$\Rightarrow F = \frac{\mu mg}{\cos\theta + \mu \sin\theta}$



$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = \frac{\frac{1}{\sqrt{3}} \times 10}{\frac{2}{\sqrt{3}}} = 5$

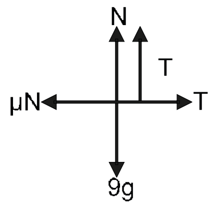
3. A boy of mass 4 kg is standing on a piece of wood having mass 5kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is ________N.

[Take g = 10 ms^{-2}]



Ans. (30)

Sol.



N + T = 90

T = μ N = 0.5 (90–T)

1.5 T = 45

T = 30

4. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area 4 cm^2 by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3}$ cm. Then the thickness of oleic acid layer will be x × 10^{-14} m. Where x is__________.

Ans. (25)

Sol. $4t_T = 100 \times \frac{4}{3}\pi r^3$

$= 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9} = 10^{-8}\ cm^3$

$t_T = 25 \times 10^{-10}$ cm

$= 25 \times 10^{-12}$ m

$t_0 = 0.01\ t_T = 25 \times 10^{-14}$ m

= 25

5. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{1/\alpha}$, where α is___________.

Ans. (3)

Sol. $F = \frac{-dU}{dr} = -4U_0 r^3 = \frac{mv^2}{r}$

$mv^2 = 4U_0 r^4$

$v \propto r^2$

$mvr = \frac{nh}{2\pi}$

$r^3 \propto n$

$r \propto n1/3$

= 3

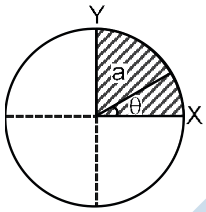
6. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the Y – Z plane is ________ Nm^2C^{-1}.

Ans. (640)

Sol. $\phi = E_x A \Rightarrow \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640$

7. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ________.

[a is an area as shown in the figure]



Ans. (4)

Sol. C.O.M of quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi} = 4$

8. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{nd}}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. the value of 'x' is__________.

Ans. (30)

Sol. $\lambda_m = \frac{\lambda_a}{\mu} \Rightarrow \mu = \frac{3}{2}$

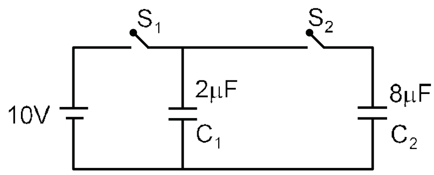
$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$

$\frac{3}{2 \times 10} + \frac{1}{15} = \frac{\frac{3}{2} - 1}{R}$

$R = \frac{30}{13}$

= 30

9. A 2 μF capacitor C_1 is first charged to a potential difference of 10 V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is_______μC.



Ans. (16)

Sol. $20 = (C_1 + C_2) V \Rightarrow V = 2$ volt.

$Q_2 = C_2V = 16\ \mu C$

$= 16$

10. Seawater at a frequency $f = 9 \times 10^2$ Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\ \Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi ft)$. Then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is _________

(Given : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$)

Ans. (6)

Sol. $J_c = \frac{E}{\rho} = \frac{V}{\rho d}$

$J_d = \frac{1}{A}\frac{dq}{dt}$

$= \frac{C}{A}\frac{dV_c}{dt}$

$= \frac{\in}{d}\frac{dV_c}{dt}$

$\Rightarrow \frac{V_0 \sin 2\pi ft}{\rho d} = 10^x \times \frac{80\varepsilon_0}{d} V_0 (2\pi f)\cos 2\pi ft$

$\tan\left(2\pi \times \frac{900}{800}\right) = 10^x \times \frac{40}{9 \times 10^9} \times 900$

$= x = 6$

PART B : CHEMISTRY

Single Choice Type

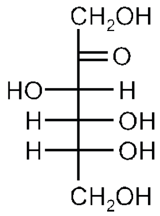
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Fructose is an example of :-

(1) Pyranose (2) Ketohexose (3) Aldohexose (4) Heptose

Ans. (2)

Sol. Fructose is a ketohexose.



2. The set of elements that differ in mutual relationship from those of the other sets is :

(1) Li – Mg (2) B – Si (3) Be – Al (4) Li – Na

Ans. (4)

Sol. Li–Mg, B–Si, Be–Al show diagonal relationship but Li and Na do not show diagonal relationship as both belongs to same group and not placed diagonally.

3. The functional groups that are responsible for the ion-exchange property of cation and anion exchange resins, respectively, are :

(1) $-SO_3H$ and $-NH_2$ (2) $-SO_3H$ and –COOH (3) $-NH_2$ and –COOH (4) $-NH_2$ and $-SO_3H$

Ans. (1)

Sol. Cation exchanger contains $-SO_3H$ or –COOH groups while anion exchanger contains basic groups like $-NH_2$.

4. Match List-I and List-II :

	List-I		List-II
(a)	Haematite	(i)	$Al_2O_3.xH_2O$
(b)	Bauxite	(ii)	Fe_2O_3
(c)	Magnetite	(iii)	$CuCO_3.Cu(OH)_2$
(d)	Malachite	(iv)	Fe_3O_4

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(3) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(4) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Ans. (4)

Sol.

	Ore	Formula
(a)	Haematite	Fe_2O_3
(b)	Bauxite	$Al_2O_3.xH_2O$
(c)	Magnetite	Fe_3O_4
(d)	Malachite	$CuCO_3.Cu(OH)_2$

5. The correct pair(s) of the ambident nucleophiles is (are) :

(A) AgCN/KCN (B) RCOOAg/RCOOK (C) $AgNO_2/KNO_2$ (D) AgI/KI

(1) (B) and (C) only (2) (A) only (3) (A) and (C) only (4) (B) only

Ans. (3)

Sol. Ambident nucleophile

(A) KCN & AgCN

(C) $AgNO_2$ & KNO_2

6. The set that represents the pair of neutral oxides of nitrogen is :

(1) NO and N_2O (2) N_2O and N_2O_3 (3) N_2O and NO_2 (4) NO and NO_2

Ans. (1)

Sol. N_2O and NO are neutral oxides of nitrogen NO_2 and N_2O_3 are acidic oxides.

7. Match List-I with List-II :

	List-I		List-II
(a)	$[Co(NH_3)_6]\,[Cr(CN)_6]$	(i)	Linkage isomerism
(b)	$[Co(NH_3)_3\,(NO_2)_3]$	(ii)	Solvate isomerism
(c)	$[Cr(H_2O)_6]Cl_3$	(iii)	Co-ordination isomerism
(d)	cis-$[CrCl_2(ox)_2]^{3-}$	(iv)	Optical isomerism

Choose the correct answer from the options given below :

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv) (2) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv) (4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Ans. (1)

Sol. Complex Type of Isomerism

(a) $[Co(NH_3)_6]\,[Cr(CN)_6]$ Co-ordination isomerism

(b) $[Co(NH_3)_3\,(NO_2)_3]$ Linkage isomerism

(c) $[Cr(H_2O)_6]Cl_3$ Solvate isomerism

(d) cis-$[CrCl_2(ox)_2]^{3-}$ Optical isomerism

8. Primary, secondary and tertiary amines can be separated using :-

(1) Para-Toluene sulphonyl chloride (2) Chloroform and KOH

(3) Benzene sulphonic acid (4) Acetyl amide

Ans. (1)

Sol. Primary amines react with Para Toluene sulfonyl chloride to form a precipitate that is soluble in NaOH.

Secondary amines reacts with para toluene sulfonyl chloride to give a precipitate that is insoluble in NaOH.

Tertiary amines do not react with para toluen.

9. The common positive oxidation states for an element with atomic number 24, are :

(1) +2 to +6 (2) +1 and +3 to +6 (3) +1 and +3 (4) +1 to +6

Ans. (1)

Sol. Cr(Z=24)

[Ar] $4s^1 3d^5$ Cr shows common oxidation states starting from +2 to +6.

10. Match List-I with List-II :

	List-I **Chemical Compound**		**List-II** **Used as**
(a)	Sucralose	(i)	Synthetic detergent
(b)	Glyceryl ester of stearic acid	(ii)	Artificial sweetener
(c)	Sodium benzoate	(iii)	Antiseptic
(d)	Bithionol	(iv)	Food preservative

Choose the correct match :

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Ans. (2)

Sol. Artificial sweetener : Sucralose

Antiseptic : Bithional

Preservative : Sodium Benzoate

Glyceryl ester of stearic acid : Sodium steasate

11. Given below are two statements :

Statement-I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.

Statement-II : n-alkanes can be easily oxidised to corresponding alcohol with $KMnO_4$.

Choose the correct option :

(1) Both statement I and statement II are correct

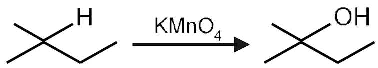
(2) Both statement I and statement II are incorrect

(3) Statement I is correct but Statement II is incorrect

(4) Statement I is incorrect but Statement II is correct

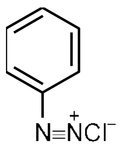
Ans. (3)

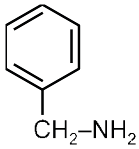
Sol. Alkane are very less reactive, tertiary hydrogen can oxidise to alcohal with $KMnO_4$.

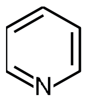


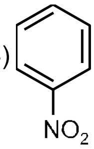
2-methyl-butane

12. Nitrogen can be estimated by Kjeldahl's method for which of the following compound ?

(1) 

(2) 

(3) 

(4) 

Ans. (2)

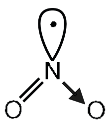
Sol. Kjeldahl method is not applicable to compounds containing nitrogen in nitro group, Azo groups and nitrogen present in the ring (e.g Pyridine) as nitrogen of these compounds does not change to Ammonium sulphate under these conditions.

13. Amongst the following, the linear species is :

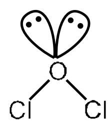
(1) NO_2 (2) Cl_2O (3) O_3 (4) N_3^-

Ans. (4)

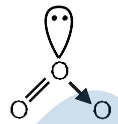
Sol.



Bent shape



Bent shape



Bent shape

$\bar{N} = \overset{+}{N} = \bar{N}$

Linear

14. $\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Enzyme A}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$

$\underset{\text{Glucose}}{C_6H_{12}O_6} \xrightarrow{\text{Enzyme B}} 2C_2H_5OH + 2CO_2$

In the above reactions, the enzyme A and enzyme B respectively are :-

(1) Amylase and Invertase (2) Invertase and Amylase

(3) Invertase and Zymase (4) Zymase and Invertase

Ans. (3)

Sol. Informative

OR

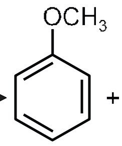
$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$

$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$

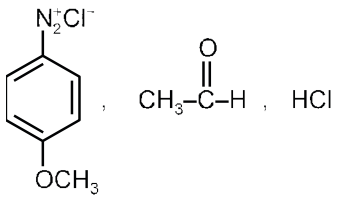
15. One of the by-products formed during the recovery of NH_3 from Solvay process is :

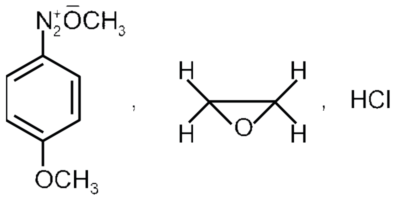
(1) $Ca(OH)_2$ (2) $NaHCO_3$ (3) $CaCl_2$ (4) NH_4Cl

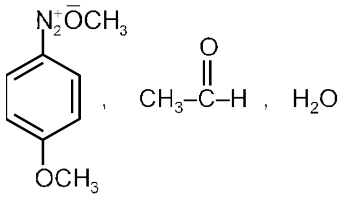
Ans. (3)

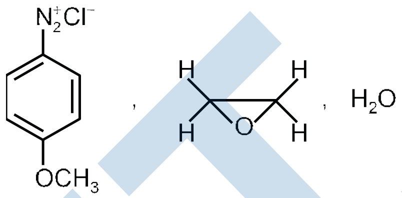
16. $C_7H_7N_2OCl + C_2H_5OH \longrightarrow$  $+ N_2 +$ "X" + "Y"
(A)

In the above reaction, the structural formula of (A), "X" and "Y" respectively are :

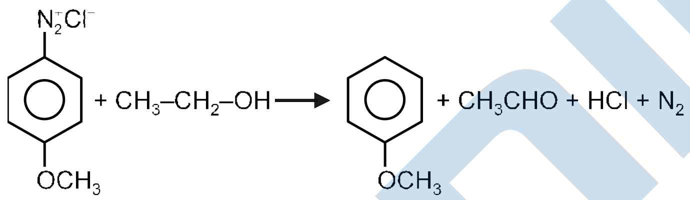
(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. 

Anisole

17. For the coagulation of a negative sol, the species below, that has the highest flocculating power is :

(1) SO_4^{2-} (2) Ba^{2+} (3) Na^{+} (4) PO_4^{3-}

Ans. (2)

Sol. To coagulate negative sol, cation with higher charge has higher coagulation value.

18. Which of the following statement(s) is (are) incorrect reason for eutrophication ?

(A) excess usage of fertilisers

(B) excess usage of detergents

(C) dense plant population in water bodies

(D) lack of nutrients in water bodies that prevent plant growth

Choose the most appropriate answer from the options given below :

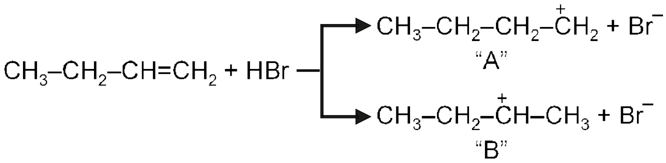
(1) (A) only (2) (C) only

(3) (B) and (D) only (4) (D) only

Ans. (4)

Sol. The process in which nutrient enriched water bodies support a dense plant population which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as eutrophication.

19. Choose the correct statement regarding the formation of carbocations A and B given :-



(1) Carbocation B is more stable and formed relatively at faster rate

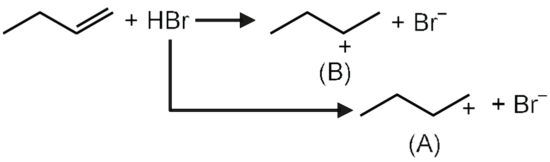
(2) Carbocation A is more stable and formed relatively at slow rate

(3) Carbocation B is more stable and formed relatively at slow rate

(4) Carbocation A is more stable and formed relatively at faster rate

Ans. (1)

Sol.



This is more stable due to secondary cation formation and formed with faster rate due to low activation energy.

20. During which of the following processes, does entropy decrease ?

(A) Freezing of water to ice at 0°C

(B) Freezing of water to ice at –10°C

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

(D) Adsorption of CO(g) and lead surface

(E) Dissolution of NaCl in water

(1) (A), (B), (C) and (D) only

(2) (B) and (C) only

(3) (A) and (E) only

(4) (A), (C) and (E) only

Ans. (1)

Sol. (A) Water $\xrightarrow{0^\circ C}$ ice; $\Delta S = -ve$

(B) Water $\xrightarrow{-10^\circ C}$ ice; $\Delta S = -ve$

(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$; $\Delta S = -ve$

(D) Adsorption; $\Delta S = -ve$

(E) $NaCl(s) \rightarrow Na^+(aq) + Cl^-(aq)$; $\Delta S = +ve$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03 Ω. The conductivity of the HCl solution is ____ $\times 10^{-2}$ S m^{-1}.

Ans. (57)

Sol. $k = \frac{1}{R} \cdot G^*$

For same conductivity cell, G* is constant and hence k.R. = constant.

$\therefore$ 0.14 × 4.19 = k × 1.03 or, k of HCl solution $= \frac{0.14 \times 4.19}{1.03}$

$= 0.5695\ Sm^{-1}$

$= 56.95 \times 10^{-2}\ Sm^{-1} \approx 57 \times 10^{-2}\ Sm^{-1}$

2. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is ____.

Ans. (6)

Sol. $Fe^{3+} + 3K^{+} + 3C_2O_4^{2-} \longrightarrow K_3[Fe(C_2O_4)_3]$

(A)

Secondary valency of Fe in 'A' is 6.

3. The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____.

Ans. (27)

Sol. Reaction : $2A + B_2 \rightarrow 2AB$

As the reaction is elementary, the rate of reaction is

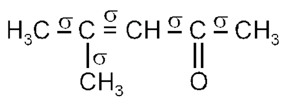
$r = K . [A]^2 [B_2]$

on reducing the volume by a factor of 3, the concentrations of A and B_2 will become 3 times and hence, the rate becomes $3^2 \times 3 = 27$ times of initial rate.

4. The total number of C–C sigma bond/s in mesityl oxide ($C_6H_{10}O$) is_____.

Ans. (5)

Sol. Mesityle oxide



$\therefore \quad C \underset{\sigma}{-} C = 5$

5. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is_____ u.

[Density of water = 1.0 g cm^{-3}]

Ans. (85)

Sol.

	$K_4\,Fe(CN)_6 \rightleftharpoons$	$4K^+$ +	$Fe(CN)_6^{4-}$
Initial conc.	1 m	0	0
Final conc.	(1 – 0.4)m	4 × 0.4	0.4m
	= 0.6 m	= 1.6 m	

Effective molality = 0.6 + 1.6 + 0.4 = 2.6m

For same boiling point, the molality of another solution should also be 2.6 m.

Now, 18.1 weight percent solution means 18.1 gm solute is present in 100 gm solution and hence, (100 – 18.1 =) 81.9 gm water.

$$\text{Now, } 2.6 = \frac{18.1/M}{81.9/1000}$$

$\therefore$ Molar mass of solute, M = 85

6. In the ground state of atomic Fe (Z = 26), the spin-only magnetic moment is ______ × 10^{-1} BM.

[Given : $\sqrt{3} = 1.73,\ \sqrt{2} = 1.41$]

Ans. (49)

Sol. $Fe \rightarrow [Ar]\,4s^2 3d^6$ $\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

Number of unpaired e^- = 4

$\mu = \sqrt{4(4+2)}$ B.M.

$\mu = \sqrt{24}$ B.M.

μ = 4.89 B.M.

$\mu = 48.9 \times 10^{-1}$ B.M.

Nearest integer value will be 49.

7. The number of chlorine atoms in 20 mL of chlorine gas at STP is_____10^{21}.

[Assume chlorine is an ideal gas at STP R = 0.083 L bar mol^{-1} K^{-1}, $N_A = 6.023 \times 10^{23}$]

Ans. (1)

Sol. PV = nRT

$$1.0 \times \frac{20}{1000} = \frac{N}{6.023 \times 10^{23}} \times 0.083 \times 273$$

$\therefore$ Number of Cl_2 molecules, N = 5.3×10^{20}

Hence, Number of Cl-atoms = 1.06×10^{21}

$\approx 1 \times 10^{21}$

8. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is _____ 10^{14}.

[Atomic Mass : K : 39.1 u, Br : 79.9 u, $N_A = 6.023 \times 10^{23}$]

Ans. (5)

Sol. 1 mole KBr (= 119 gm) have $\frac{10^{-5}}{100}$ moles $SrBr_2$ and hence, 10^{-7} moles cation vacancy (as 1 Sr^{2+} will result 1 cation vacancy)

$\therefore$ Required number of cation vacancies

$$= \frac{10^{-7} \times 6.023 \times 10^{23}}{119} = 5.06 \times 10^{14} \approx 5 \times 10^{14}$$

9. Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$.

The temperature at which $K_C = 20.4$ and $K_P = 600.1$, is_____K.

[Assume all gases are ideal and R = 0.0831 L bar K^{-1} mol^{-1}]

Ans. (354)

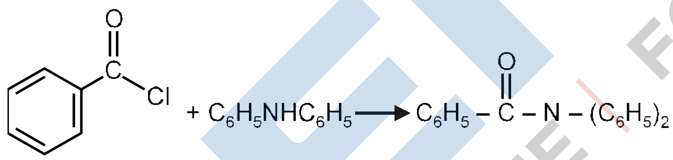
Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$; $\Delta n_g = 2 - 1 = 1$

Now, $K_p = K_c \cdot (RT)^{\Delta ng}$

or, $600.1 = 20.4 \times (0.0831 \times T)^1$

$\therefore$ T = 353.99 K = 354K

10.



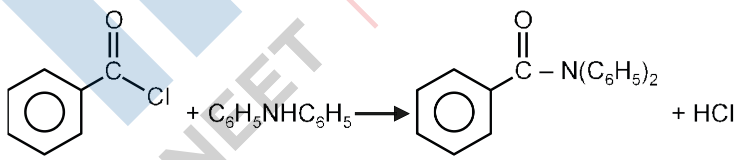
0.140 g   0.388 g   0.210 g

Consider the above reaction. The percentage yield of amide product is ______.

(Given : Atomic mass : C : 12.0 u, H : 1.0u, N : 14.0 u, O : 16.0 u, Cl : 35.5 u)

Ans. (77)

Sol.



1 mole	1 mole	1 mole
= 140.5 gm	= 169 gm	= 273 gm

$\therefore$ 0.140 gm $\quad \frac{169}{140.5} \times 0.140$

L.R. $\quad$ = 0.168 gm < 0.388 gm

excess

∴ Theoretical amount of given product formed

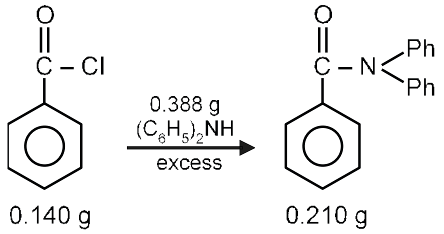
$$= \frac{273}{140.5} \times 0.140 = 0.272 \text{ gm}$$

But its actual amount formed is 0.210 gm.

Hence, the percentage yield of product.

$$= \frac{0.210}{0.272} \times 100 = 77.20 \approx 77$$

OR



$$\text{Mole of Ph} - \text{CoCl} = \frac{0.140}{140} = 10^{-3} \text{ mol}$$

Mole of $Ph - \overset{O}{\overset{\|}{C}} - N(Ph)_2$, that should be obtained by mol-mol analysis = 10^{-3} mol.

Theoretical mass of product = $10^{-3} \times 273 = 273 \times 10^{-3}$ g

Observed mass of product = 210×10^{-3} g

$$\%\text{yield of product} = \frac{210 \times 10^{-3}}{273 \times 10^{-3}} \times 100 = 76.9\% = 77$$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let f : R → R be defined as $f(x) = e^{-x} \sin x$. If F : [0, 1] → R is a differentiable function such that $F(x) = \int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x) + f(x)e^x\,dx$ lies in the interval

(1) $\left[\frac{327}{360}, \frac{329}{360}\right]$ (2) $\left[\frac{330}{360}, \frac{331}{360}\right]$ (3) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (4) $\left[\frac{335}{360}, \frac{336}{360}\right]$

Ans. (2)

Sol. $f(x) = e^{-x} \sin x$

Now, $F(x) = \int_0^x f(t)dt \quad \Rightarrow F'(x) = f(x)$

$$I = \int_0^1 (F'(x) + f(x))e^x\,dx = \int_0^1 (f(x) + f(x)) \cdot e^x\,dx$$

$$= 2\int_0^1 f(x) \cdot e^x\,dx = 2\int_0^1 e^{-x} \sin x \cdot e^x\,dx$$

$$= 2\int_0^1 \sin x\,dx$$

= 2(1 – cos 1)

$$I = \left\{1 - \left(1 - \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \frac{1}{\lfloor 8} ....\right)\right\}$$

$$I = 1 - \frac{2}{\lfloor 4} + \frac{2}{\lfloor 6} + \frac{2}{\lfloor 9} ....$$

$$I - \frac{2}{\lfloor 4} < I < 1 - \frac{2}{\lfloor 4} + \frac{2}{\lfloor 6}$$

$$\frac{11}{12} < I < \frac{331}{360}$$

$$\Rightarrow I \in \left[\frac{11}{20}, \frac{331}{360}\right]$$

$$\Rightarrow I \in \left[\frac{330}{360}, \frac{331}{360}\right]$$

2. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta e^{\frac{1}{2}} + \gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

(1) 0 (2) 20 (3) 25 (4) 10

Ans. (1)

Sol. Let $I = \int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \int_0^{10} \frac{[\sin 2\pi x]}{e^{[x]}}dx$

Function $f(x) = \frac{[\sin 2\pi x]}{e^{[x]}}$ is periodic with period '1'

Therefore

$$I = 10\int_0^1 \frac{[\sin 2\pi x]}{e^{[x]}}dx$$

$$= 10\int_0^1 \frac{[\sin 2\pi x]}{e^{x}}dx$$

$$= 10\left(\int_0^{1/2} \frac{[\sin 2\pi x]}{e^{x}}dx + \int_{1/2}^{1} \frac{[\sin 2\pi x]}{e^{x}}dx\right)$$

$$= 10\left(0 + \int_{1/2}^{1} \frac{(-1)}{e^{x}}dx\right)$$

$$= -10\int_{1/2}^{1} e^{-x}\,dx$$

$= 10\,(e^{-1} - e^{-1/2})$

Now,

$10 \cdot e^{-1} - 10 \cdot e^{-1/2} = \alpha e^{-1} + \beta e^{-1/2} + \gamma$ (given)

$\Rightarrow \alpha = 10,\ \beta = -10,\ \gamma = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

3. Let y = y(x) be the solution of the differential equation

cosx (3sinx + cosx + 3)dy = (1 + y sinx (3sinx + cosx + 3))dx,

$0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$ (3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

Ans. (2)

Sol. cosx(3sin x + cosx + 3)dy = (1 + ysin x(3sin x + cosx + 3))dx

$$\frac{dy}{dx} - (\tan x)y = \frac{1}{(3\sin x + \cos x + 3)\cos x}$$

$$\text{I.F.} = e^{\int -\tan x\,dx} = e^{\ell n|\cos x|} = |\cos x|$$

$$= \cos x \ \forall\ x \in \left[0, \frac{\pi}{2}\right)$$

Solution of D.E.

$$y(\cos x) = \int (\cos x)\cdot\frac{1}{\cos x(3\sin x + \cos x + 3)}dx + C$$

$$y(\cos x) = \int \frac{dx}{3\sin x + \cos x + 3}dx + C$$

$$y(\cos x) = \int \frac{\left(\sec^2\frac{x}{2}\right)}{2\tan^2\frac{x}{2} + 6\tan\frac{x}{2} + 4}dx + C$$

Now

$$\text{Let } I_1 = \int \frac{\left(\sec^2\frac{x}{2}\right)}{2\left(\tan^2\frac{x}{2} + 3\tan\frac{x}{2} + 2\right)}dx + C$$

$$\text{Put } \tan\frac{x}{2} = 1 \Rightarrow \qquad \frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I_1 = \int \frac{dt}{t^2 + 3t + 2} = \int \frac{dt}{(t+2)(t+1)}$$

$$= \int \left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$$

$$= \ell n\left|\left(\frac{t+1}{t+2}\right)\right| = \ell n\left|\left(\frac{\tan\frac{x}{2} + 1}{\tan\frac{x}{2} + 2}\right)\right|$$

So solution of D.E.

$$y(\cos x) = \ell n\left|\frac{1 + \tan\frac{x}{2}}{2 + \tan\frac{x}{2}}\right| + C$$

$$\Rightarrow y(\cos x) = \ell n\left(\frac{1 + \tan\frac{x}{2}}{2 + \tan\frac{x}{2}}\right) + C \quad \text{for } 0 \le x < \frac{\pi}{2}$$

Now, it is given y(0) = 0

$$\Rightarrow 0 = \ell n\left(\frac{1}{2}\right) + C \qquad \Rightarrow \boxed{C = \ell n 2}$$

$$\Rightarrow y(\cos x) = \ell n\left(\frac{1+\tan\frac{x}{2}}{2+\tan\frac{x}{2}}\right) + \ell n\, 2$$

For $x = \frac{\pi}{3}$

$$y\left(\frac{1}{2}\right) = \ell n\left(\frac{1+\frac{1}{\sqrt{3}}}{2+\frac{1}{\sqrt{3}}}\right) + \ell n 2$$

$$y = 2\,\ell n\left(\frac{2\sqrt{3}+10}{11}\right)$$

4. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

(1) 1124 (2) 1324 (3) 1024 (4) 924

Ans. (4)

Sol. $\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$

$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \ldots\ldots + {}^6C_6 \cdot {}^6C_0$

Now, $(1+x)^6\,(1+x)^6$

$= ({}^6C_0 + {}^6C_1\,x + {}^6C_2\,x^2 + \ldots\ldots + {}^6C_6\,x^6)\,({}^6C_0 + {}^6C_1\,x + {}^6C_2\,x^2 + \ldots\ldots + {}^6C_6\,x^6)$

Comparing coefficient of x^6 both sides

${}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + \ldots\ldots + {}^6C_6 \cdot {}^6C_0 = {}^{12}C_6$

$= 924$

5. The value of $\lim_{n\to\infty}\frac{[r]+[2r]+\ldots..+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\frac{r}{2}$ (2) r (3) 2r (4) 0

Ans. (1)

Sol. We know that

$r \le [r] < r + 1$

and $2r \le [2r] < 2r + 1$

$3r \le [3r] < 3r + 1$

$\vdots \quad \vdots \quad \vdots$

$nr \le [nr] < nr + 1$

$r + 2r + \ldots.+ nr \le [r] + [2r] + \ldots.+ [nr] < (r + 2r + \ldots.+ nr) + n$

$$\frac{\frac{n(n+1)}{2}\cdot r}{n^2} \le \frac{[r]+[2r]+.....+[nr]}{n^2} < \frac{\frac{n(n+1)}{2}r+n}{n^2}$$

Now, $\lim\limits_{n\to\infty}\frac{n(n+1)\cdot r}{2\cdot n^2}=\frac{r}{2}$ and $\lim\limits_{n\to\infty}\frac{\frac{n(n+1)r}{2}+n}{n^2}=\frac{r}{2}$

So, by Sandwich Theorem, we can conclude that

$$\lim_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}=\frac{r}{2}$$

6. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x\in[-1, 1]$, and $[x]$ denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0 (3) 4 (4) Infinite

Ans. (2)

Sol. Given equation

$$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$$

Now, $\sin^{-1}\left[x^2+\frac{1}{3}\right]$ is defined if

$$-1\le x^2+\frac{1}{3}<2 \Rightarrow \frac{-4}{3}\le x^2<\frac{5}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{5}{3}} \quad ....(1)$$

and $\cos^{-1}\left[x^2-\frac{2}{3}\right]$ is defined if

$$-1\le x^2-\frac{2}{3}<2 \Rightarrow \frac{-1}{3}\le x^2<\frac{8}{3}$$

$$\Rightarrow \boxed{0\le x^2<\frac{8}{3}} \quad ....(2)$$

So, form (1) and (2) we can conclude

$$\boxed{0\le x^2<\frac{5}{3}}$$

Case - I if $0\le x^2<\frac{2}{3}$

$\sin^{-1}(0)+\cos^{-1}(-1)=x^2$

$\Rightarrow x+\pi=x^2$

$\Rightarrow x^2=\pi$

but $\pi\notin\left[0,\frac{2}{3}\right)$

$\Rightarrow$ No value of 'x'

Case - II if $\frac{2}{3} \le x^2 < \frac{5}{3}$

$\sin^{-1}(1) + \cos^{-1}(0) = x^2$

$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$

$\Rightarrow x^2 = \pi$

but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

$\Rightarrow$ No value of 'x'

So, number of solutions of the equation is zero.

7. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Ans. (4)

Sol.

	1	0	0	1
	odd place	even place	odd place	even place
or	1	0	0	1
	even place	odd place	even place	odd place

$\Rightarrow \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right) + \left(\frac{2}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$

$\Rightarrow \frac{1}{9}$

8. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval [0, 2π] is :

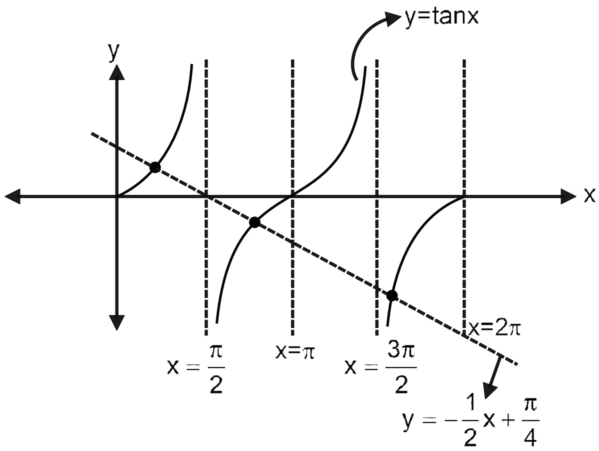
(1) 3 (2) 4 (3) 2 (4) 5

Ans. (1)

Sol. $x + 2\tan x = \frac{\pi}{2}$

$\Rightarrow 2\tan x = \frac{\pi}{2} - x$

$\Rightarrow \tan x = -\frac{1}{2}x + \frac{\pi}{4}$



Number of solutions of the given equation is '3'.

9. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : Re((1 - i)z) \ge 1\}$

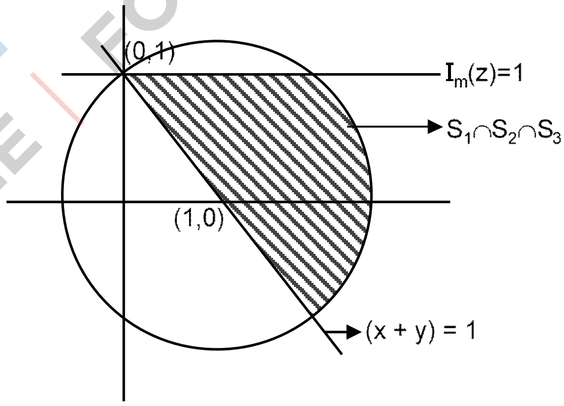
$S_3 = \{z \in \mathbb{C} : Im(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(1) is a singleton
(2) has exactly two elements
(3) has infinitely many elements
(4) has exactly three elements

Ans. (3)

Sol. For $|z-1| \le \sqrt{2}$, z lies on and inside the circle of radius $\sqrt{2}$ units and centre (1, 0).



For S_2

Let z = x + iy

Now, $(1 - i)(z) = (1 - i)(x + iy)$

$Re((1 - i)z) = x + y$

$\Rightarrow x + y \ge 1$

$\Rightarrow S_1 \cap S_2 \cap S_3$ has infinity many elements

10. If the curve y = y(x) is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}\,dx$, x > 0 which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to :

(1) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$
(2) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$
(3) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$
(4) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

Ans. (3)

Sol. $\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$

$$IF = e^{-\int \frac{dx}{2d}} = e^{-\frac{1}{2}\ln x} = \frac{1}{x^{1/2}}$$

$$y \cdot x^{-1/2} = \int \frac{x^{9/4} \cdot x^{-1/2}}{x^{5/4}\left(x^{3/4}+1\right)} dx$$

$$\int \frac{x^{1/2}}{(x^{3/4}+1)} dx$$

$x = t^4 \Rightarrow dx = 4t^3\, dt$

$$\int \frac{t^2 \cdot 4t^3\, dt}{(t^3+1)}$$

$$4\int \frac{t^2\left(t^3+1-1\right)}{(t^3+1)} dt$$

$$4\int t^2\, dt - 4\int \frac{t^2}{t^3+1} dt$$

$$\frac{4t^3}{3} - \frac{4}{3}\ln(t^3+1) + C$$

$$y x^{-1/2} = \frac{4x^{3/4}}{3} - \frac{4}{3}\ln(x^{3/4}+1) + C$$

$$1 - \frac{4}{3}\log_e 2 = \frac{4}{3} - \frac{4}{3}\log_e 2 + C$$

$$\Rightarrow C = -\frac{1}{3}$$

$$y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\ln(x^{3/4}+1) - \frac{\sqrt{x}}{3}$$

$$y(16) = \frac{4}{3} \times 32 - \frac{4}{3} \times 4\ln 9 - \frac{4}{3}$$

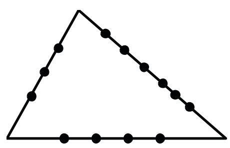
$$= \frac{124}{3} - \frac{32}{3}\ln 3 = 4\left(\frac{31}{3} - \frac{8}{3}\ln 3\right)$$

11. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 364 (2) 240 (3) 333 (4) 360

Ans. (3)

Sol. Total Number of triangles formed



$= {}^{14}C_3 - {}^{3}C_3 - {}^{5}C_3 - {}^{6}C_3$

$= 333$

12. If x, y, z are in arithmetic progression with common difference d, $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(1) 72 (2) 12 (3) 36 (4) 6

Ans. (1)

Sol. $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$R_2 \to R_1 + R_3 - 2R_2$

$\Rightarrow \begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k-6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$

$\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$

if $3z - 5x = 0 \Rightarrow 3(x + 2d) - 5x = 0$

$\Rightarrow x = 3d$ (Not possible)

$\Rightarrow 6\sqrt{2}$ $\qquad \Rightarrow k^2 = 72$

13. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, x, y $\in R$, x > 0, be such that $\overrightarrow{OP} = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in R$, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

Ans. (2)

Sol. $\overrightarrow{OP} \perp \overrightarrow{OQ}$

$\Rightarrow -x + 2y - 3x = 0$

$\Rightarrow y = 2x$ .....(i)

$|\overrightarrow{OP}|^2 = 20$

$\Rightarrow (x + 1)^2 + (y - 2)^2 + (1 + 3x)^2 = 20$

$\Rightarrow x = 1$

$\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{QR}$ are coplanar.

$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1(-z - 6) = 0$

$\Rightarrow z = -2$

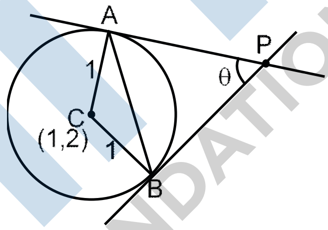
$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$

14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

Ans. (2)

Sol. $\tan\theta = \frac{12}{5}$



$PA = \cot\frac{\theta}{2}$

$\therefore$ area of $\Delta PAB = \frac{1}{2}(PA)^2 \sin\theta = \frac{1}{2}\cot^2\frac{\theta}{2}\sin\theta$

$= \frac{1}{2}\left(\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta$

$= \frac{1}{2}\left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)\left(\frac{12}{13}\right) = \frac{1}{2}\frac{18}{18}\times\frac{2}{13} = \frac{27}{26}$

area of $\Delta CAB = \frac{1}{2}\sin\theta = \frac{1}{2}\left(\frac{12}{13}\right) = \frac{6}{13}$

$\therefore \frac{\text{area of } \Delta PAB}{\text{area of } \Delta CAB} = \frac{9}{4}$

15. Consider the function $f : R \to R$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x| & , x \neq 0 \\ 0 & , x = 0 \end{cases}$. Then f is :

(1) monotonic on $(-\infty, 0) \cup (0, \infty)$
(2) not monotonic on $(-\infty, 0)$ and $(0, \infty)$
(3) monotonic on $(0, \infty)$ only
(4) monotonic on $(-\infty, 0)$ only

Ans. (2)

Sol. $f(x)=\begin{cases}-x\left(2-\sin\left(\frac{1}{x}\right)\right) & x<0\\ 0 & x=0\\ x\left(2-\sin\left(\frac{1}{x}\right)\right) & \end{cases}$

$f'(x)=\begin{cases}-\left(2-\sin\frac{1}{x}\right)-x\left(-\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)\right) & x<0\\ \left(2-\sin\frac{1}{x}\right)+x\left(-\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)\right) & x>0\end{cases}$

$f'(x)=\begin{cases}-2+\sin\frac{1}{x}-\frac{1}{x}\cos\frac{1}{x} & x<0\\ 2-\sin\frac{1}{x}+\frac{1}{x}\cos\frac{1}{x} & x>0\end{cases}$

f'(x) is an oscillating function which is non-monotonic in $(-\infty, 0)\cup(0, \infty)$.

16. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to :

(1) 11 (2) 14 (3) 16 (4) 20

Ans. (2)

Sol. Tangent to parabola

$2y = 2(x + 6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$16 = 2(1)^2 + b$

$\Rightarrow b = 14$

17. The value of the limit $\lim_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) 0 (4) $\frac{1}{4}$

Ans. (1)

Sol. $\lim_{\theta\to 0}\frac{\tan(\pi(1-\sin^2\theta))}{\sin(2\pi\sin^2\theta)}$

$=\lim_{\theta\to 0}\frac{-\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}$

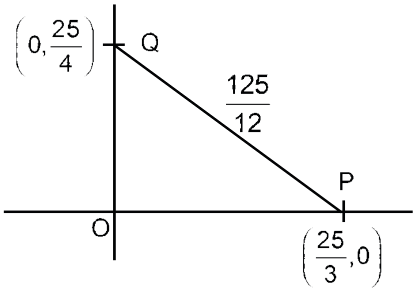
$=\lim_{\theta\to 0}-\left(\frac{\tan(\pi\sin^2\theta)}{\pi\sin^2\theta}\right)\left(\frac{2\pi\sin^2\theta}{\sin(2\pi\sin^2\theta)}\right)\times\frac{1}{2}=-\frac{1}{2}$

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

Ans. (3)

Sol. Tangent to circle 3x + 4y = 25



OP + OQ + OR = 25

$$\text{Incentre} = \left(\frac{\frac{25}{4}\times\frac{25}{3}}{25}, \frac{\frac{25}{4}\times\frac{25}{3}}{25}\right)$$

$$= \left(\frac{25}{12}, \frac{25}{12}\right)$$

$$\therefore r^2 = 2\left(\frac{25}{12}\right)^2 = 2\times\frac{625}{144} = \frac{625}{72}$$

19. If the Boolean expression $(p \wedge q) * (p \times q)$ is a tautology, then $*$ and $\times$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

Ans. (1)

Sol. **Option (1)**

$(p \wedge q) \longrightarrow (p \rightarrow q)$

$= \sim(p \wedge q) \vee (\sim p \vee q)$

$= (\sim p \vee \sim q) \vee (\sim p \vee q)$

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee t = t$

Option (2)

$(p \wedge q) \wedge (p \vee q) = (p \wedge q)$ (Not a tautology)

Option (3)

$(p \wedge q) \vee (p \rightarrow q)$

$= (p \wedge q) \vee (\sim p \vee q)$

$= \sim p \vee q$ (Not a tautology)

Option (4)

$(p \wedge q) \wedge (p \rightarrow q)$

$= (p \wedge q) \wedge (\sim p \vee q)$

$= p \wedge q$ (Not a tautology)

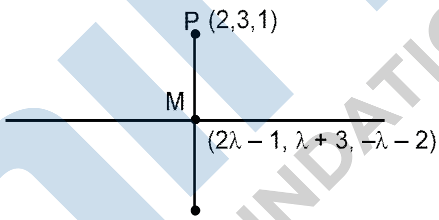
Option (1)

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

Ans. (2)

Sol. Line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$



$\overrightarrow{PM} = (2\lambda - 3, \lambda, -\lambda - 3)$

$\overrightarrow{PM} \perp (2\hat{i} + \hat{j} - \hat{k})$

$4\lambda - 6 + \lambda + \lambda + 3 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore M \equiv \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore$ Reflection (–2, 4, –6)

Plane : $\begin{vmatrix} x-2 & y-1 & z+1 \\ 3 & -2 & 1 \\ 4 & -3 & 5 \end{vmatrix} = 0$

$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-1) = 0$

$\Rightarrow -7x + 14 - 11y + 11 - z - 1 = 0$

$\Rightarrow 7x + 11y + z = 24$

$\therefore \alpha = 7, \beta = 11, \gamma = 1$

$\alpha + \beta + \gamma = 19$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to :

Ans. (2)

Sol. $2\log_{10}(4^x - 2) = 1 + \log_{10}\left(4^x + \frac{18}{5}\right)$

$(4^x - 2)^2 = 10\left(4^x + \frac{18}{5}\right)$

$(4^x)^2 + 4 - 4(4^x) - 32 = 0$

$(4^x - 16)(4^x + 2) = 0$

$4^x = 16$

$x = 2$

$\begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 3(-2) - 1(0-4) + 4(1)$

$= -6 + 4 + 4 = 2$

2. Let f : [– 1, 1] → R be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where a, b, c ∈ R such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to _______.

Ans. (5)

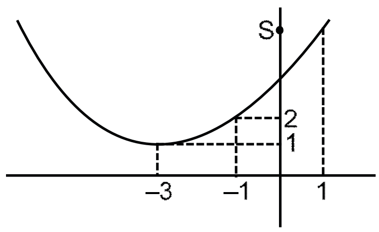
Sol. $f : [-1, 1] \to R$

$f(x) = ax^2 + bx + c$

$f(-1) = a - b + c = 2$ ...(1)

$f'(-1) = -2a + b = 1$ ...(2)

$f''(x) = 2a$



$\Rightarrow$ Max. value of $f''(x) = 2a = \frac{1}{2}$

$\Rightarrow a = \frac{1}{2};\ b = \frac{3}{2};\ c = \frac{13}{4}$

$\therefore\ f(x)=\frac{x^2}{4}+\frac{3}{2}x+\frac{13}{4}$

For, $x \in [-1, 1] \Rightarrow 2 \le f(x) \le 5$

$\therefore$ Least value of α is 5

3. Let $f : [-3, 1] \to R$ be given as

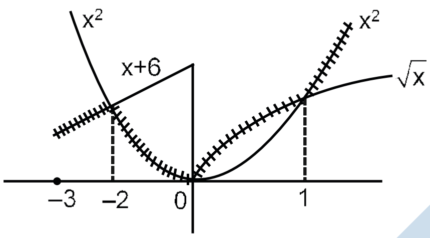
$$f(x)=\begin{cases}\min\{(x+6), x^2\}, & -3 \le x \le 0\\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1\end{cases}$$

If the area bounded by y = f(x) and x-axis is A, then the value of 6A is equal to ______.

Ans. (41)

Sol. $f : [-3, 1] \to R$

$$f(x)=\begin{cases}\min\{(x+6), x^2\}, & -3 \le x \le 0\\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1\end{cases}$$



area bounded by y = f(x) and x-axis

$$=\int_{-3}^{-2}(x+6)\,dx+\int_{-2}^{0}x^2\,dx+\int_{0}^{1}\sqrt{x}\,dx$$

$$A=\frac{41}{6}$$

6A = 41

4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \ne \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha+\cos 3\beta+\cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)$ is equal to :

Ans. (144)

Sol. Since orthocentre and circumcentre both lies on y-axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow$ $\Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow$ $\cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}$$

$$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma}$$

= 12

5. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

Ans. (68)

Sol. Let number be $a_1, a_2, a_3, \ldots\ldots a_{2n}, b_1, b_2, b_3 \ldots . b_n$

$$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2$$

$$\Rightarrow \sum a^2 + \sum b^2 = 87n$$

Now, distribution becomes

$a_1 + 1, a_2 + 1, a_3 + 1, \ldots\ldots a_{2n} + 1, b_1 - 1, b_2 - 1 \ldots . b_n - 1$

Variance

$$= \frac{\sum (a+1)^2 + \sum (b-1)^2}{3n} - \left(\frac{12n + 2n + 3n - n}{3n}\right)^2$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 3n + 2(12n) - 2(3n)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2$$

$\Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$

6. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _______.

Ans. (4)

Sol. $T_{r+1} = {}^nC_r (x)^{n-r} \left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, a^r\, x^{n-3r}$

${}^nC_2\, a^2 : {}^nC_3\, a^3 : {}^nC_4\, a^4 = 12 : 8 : 3$

After solving

$n = 6,\quad a = \frac{1}{2}$

For term independent of 'x' $\Rightarrow n = 3r$

$r = 2$

$\therefore$ Coefficient is ${}^6C_2\left(\frac{1}{2}\right)^2 = \frac{15}{4}$

Nearest integer is 4.

7. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ________.

Ans. (2020)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

AB = B

$\Rightarrow (A - I)\,B = O$

$\Rightarrow |A - I| = O$, since $B \neq O$

$\begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$

ad – bc = 2020

8. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ________.

Ans. (486)

Sol. Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)

$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$

Since $\vec{x} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$3\lambda + 8\mu = 0$ .....(1)

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$

$\frac{\vec{x} \cdot \vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$

$6\lambda - \mu = 51$ .....(2)

From (1) and (2)

$\lambda = 8,\ \mu = -3$

$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$

$|\vec{x}|^2 = 486$

9. Let $I_n = \int_1^e x^{19}(\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equal to _______.

Ans. (1)

Sol. $I_n = \int_1^e x^{19}(\log|x|)^n dx$

$$= \left[\frac{x^{20}}{20}(\ell n|x|)^n\right]_1^e - \int_1^e n\frac{(\ell n+1)^{n-1}}{x}\cdot\frac{x^{20}}{20}dx$$

$$\Rightarrow I_n = \frac{e^{20}}{20} - \frac{n}{20}\int_1^e (\ell n|x|)^{n-1}\cdot x^{19}dx$$

$$\Rightarrow I_n = \frac{e^{20}}{20} - \frac{n}{20}I_{n-1}$$

$\Rightarrow 20I_n + nI_{n-1} = e^{20}$

Put n = 10 and n = 9

$20I_{10} + 10I_9 = e^{20}$ ....(1)

$20I_9 + 9I_8 = e^{20}$ ....(2)

(1) – (2)

$\Rightarrow 20I_{10} - 10I_9 - 9I_8 = 0$

$\Rightarrow 20I_{10} = 10I_9 + 9I_8$

$\Rightarrow \alpha = 10,\ \beta = 9$

$\alpha - \beta = 1$

10. Let P be an arbitrary point having sum of the squares of the distance from the planes x + y + z = 0, $\ell x - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $\ell - n$ is equal to _______.

Ans. (0)

Sol. Let point P is (α, β, γ)

$$\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2 + \left(\frac{\ell\alpha - n\gamma}{\sqrt{\ell^2+n^2}}\right)^2 + \left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2 = 9$$

Locus is

$$\frac{(x+y+z)^2}{3}+\frac{(\ell x-nz)^2}{\ell^2+n^2}+\frac{(x-2y+z)^2}{6}=9$$

$$x^2\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)+y^2+z^2\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)+2zx\left(\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}\right)-9=0$$

Since its given that $x^2 + y^2 + z^2 = 9$

After solving $\ell = n$